Datashare 6
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Geometry and Kinematics of Single-Layer Detachment Folds
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This datashare presents an appendix describing the equations in the article.

APPENDIX

Equation (2) to describe the kinematics of any detachment fold

From angular relationships we know that (Figure 21):

$$Lb + Lt + Lf = S + Lb\cos(\vartheta b) + Lt + Lf\cos(\vartheta f) \qquad (21).$$

Simplifying:

$$Lb + Lf = S + Lb\cos(\vartheta b) + Lf\cos(\vartheta f) \qquad (22).$$

Rearranging we obtain equation (2).

Equation (6) to describe the kinematics of detachment folds formed according to the Dahlstrom (1990) model

From angular relationships we know that the area of the core of a detachment fold (Figure 21) is:

$$\frac{uLb\cos(\vartheta b)}{2} + \frac{uLf\cos(\vartheta f)}{2} + uLt \qquad (23).$$

Substituting the uplift from equation (1):

$$\frac{Lb^2\cos(\vartheta b)\sin(\vartheta b)}{2} + \frac{Lf^2\cos(\vartheta f)\sin(\vartheta f)}{2} + LtLf\sin(\vartheta f) \qquad (24).$$

Equating this expression to the shortening times the depth to detachment we obtain equation (6).

Equation (7) to describe the kinematics of detachment folds formed according to the Blay et al. (1977) model

From angular relationships we know (Figure 21):

$$\frac{\sin((\vartheta b/2)+(\vartheta f/2))}{Lb\cos(\vartheta b)+Lt+Lf\cos(\vartheta f)} = \frac{\sin(90^o-(\vartheta b/2))}{b} \quad (25).$$

Rearranging the equation gives:

$$b = \frac{\sin(90^o-(\vartheta b/2))(Lb\cos(\vartheta b)+Lt+Lf\cos(\vartheta f))}{\sin((\vartheta b/2)+(\vartheta f/2))} \quad (26).$$

From angular relationships we also know:

$$z = b\sin(90^o-(\vartheta f/2) \quad (27).$$

Substituting equation (26) into equation (27):

$$z = \frac{\sin(90^o-(\vartheta b/2))\sin(90^o-(\vartheta f/2))(Lb\cos(\vartheta b)+Lt+Lf\cos(\vartheta f))}{\sin((\vartheta b/2)+(\vartheta f/2))} \quad (28).$$

Applying the equation of sinus of addition of two angles:

$$z = \frac{(\sin 90^o\cos(\vartheta f/2)-\cos 90^o\sin(\vartheta f/2))(\sin 90^o\cos(\vartheta b/2)-\cos 90^o\sin(\vartheta b/2))}{\sin((\vartheta b/2)+(\vartheta f/2))} \underline{(Lb\cos(\vartheta b)+Lt+Lf\cos(\vartheta f))} \quad (29).$$

Simplifying we obtain equation (7).

Equation (8) to describe the maximum fold core area of a detachment fold formed according to De Sitter (1956) model

The area vs. shortening function in a De Sitter type detachment fold starts at area equal to zero, increases to a maximum and then decreases to zero again. We know that at maxima, the first derivative of a function is zero. Therefore, to know the point where the fold core area is maxima, we have to find the first derivative of the area function and equate this function to zero. The equation that describes the fold core area is equation (24). This equation is expressed as a function of Lb, Lf, Lt, ϑb and ϑf. In a De Sitter type detachment fold, Lb, Lf and Lt are constant values, whereas ϑb and ϑf are unknown. Firstly, ϑb will be expressed as a function of ϑf in order to have only one unknown in equation (24). Rearranging equation (1):

$$\sin(\vartheta b) = \frac{Lf}{Lb\sin(\vartheta f)} \qquad (30).$$

From trigonometric relationships we also know that:

$$\sin^2(\vartheta b) + \cos^2(\vartheta b) = 1 \qquad (31).$$

Rearranging this equation:

$$\cos(\vartheta b) = (1 - \sin^2(\vartheta b))^{1/2} \qquad (32).$$

Substituting equation (30) into equation (32):

$$\cos(\vartheta b) = (1 - Lf^2 / Lb^2 \sin^2(\vartheta f))^{1/2} \qquad (33).$$

Substituting equations (30) and (33) into equation (24) we obtain the fold core area (A) as a function of ϑf:

$$A = \frac{Lb^2(1 - (Lf^2 / Lb^2)\sin^2(\vartheta f))^{1/2}(Lf / Lb\sin(\vartheta f))}{2} + \frac{Lf^2(\cos(\vartheta f)\sin(\vartheta f))}{2}$$
$$+ LtLf\sin(\vartheta f) \qquad (34).$$

To find the first derivative of equation (34) we will subdivide the equation in three parts separated by additions.

The derivative of the first part of equation (34) is:

$$\frac{d[(Lb^2 / 2)(1 - (Lf^2 / Lb^2)\sin^2(\vartheta f))^{1/2}(Lf / Lb\sin(\vartheta f))]}{d\vartheta f} =$$
$$\frac{Lb^2(1 - (Lf^2 / Lb^2)\sin^2(\vartheta f))^{1/2}(Lf / Lb\cos(\vartheta f))}{2}$$

$$-\frac{Lb^2(1-(Lf^2/Lb^2)\sin^2(\vartheta f))^{-1/2}(\sin(\vartheta f)\cos(\vartheta f))(Lf/Lb\sin(\vartheta f))}{2} \quad (35).$$

Simplifying:

$$\frac{d[(Lb^2/2)(1-(Lf^2/Lb^2)\sin^2(\vartheta f))^{1/2}(Lf/Lb\sin(\vartheta f))]}{d\vartheta f} =$$

$$\frac{(Lf\cos(\vartheta f))[(Lb(1-(Lf^2/Lb^2)\sin^2(\vartheta f))^{1/2})-((Lb^2/Lb)(1-(Lf^2/Lb^2)\sin^2(\vartheta f))^{-1/2}}{2}$$

$$\underline{(\sin^2(\vartheta f)))]} \quad (36).$$

The derivative of the second part of equation (34) is:

$$\frac{d[(Lf^2/2)(\cos(\vartheta f)\sin(\vartheta f))]}{d\vartheta f} = \frac{Lf^2(\cos(\vartheta f)\cos(\vartheta f))}{2} - \frac{Lf^2(\sin(\vartheta f)\sin(\vartheta f))}{2} \quad (37).$$

Simplifying:

$$\frac{d[(Lf^2/2)(\cos(\vartheta f)\sin(\vartheta f))]}{d\vartheta f} = \frac{Lf^2(\cos^2(\vartheta f)\sin^2(\vartheta f))}{2} \quad (38).$$

The derivative of the last part of equation (34) is:

$$\frac{d[LtLf\sin(\vartheta f)]}{d\vartheta f} = LtLf\cos(\vartheta f) \quad (39).$$

Adding equation (36) plus equation (38) plus equation (39) we obtain the first derivative of the area vs. shortening function, which equated to zero gives us equation (8).

Development of equation (17) to asses the angular shear of a detachment fold without flat top and with forelimb thinning or thickening

To estimate the angular shear in the case of a detachment fold without a flat top and with forelimb thickness variation, several preliminary equations must be developed. The global angular layer-parallel shear for an individual fold is equal to the shear in the forelimb, minus the shear in the hinge, plus the shear in the backlimb.

A) Shear in the hinge

The angular shear in the hinge zone is given by (Figure 22):

$$\psi = \tan^{-1}[\frac{t' \tan(90^o - \gamma **) + \tan(90^o - \gamma *)}{t}] \quad (40).$$

From internal angular relationships we know that (Figure 22):

$$\frac{t}{\cos(90^o - \gamma *)} = \frac{t'}{\cos(90^o - \gamma **)} \quad (41).$$

Using the formulas of addition of two angles this equation can be expressed as:

$$\cos 90^o \cos\gamma ** + \sin 90^o \sin\gamma ** = \frac{t' (\cos 90^o \cos\gamma * + \sin 90^o \sin\gamma *)}{t} \quad (42),$$

and finally as:

$$\gamma ** = \sin^{-1}[\frac{t' \sin\gamma *}{t}] \quad (43).$$

Replacing equation (43) into general equation (40) we obtain the angular shear in the hinge zone as a function of the thickness and γ*:

$$\psi = \tan^{-1}[\frac{t' \tan(90^o - (\sin^{-1}[t' / t \sin\gamma *]))}{t} + \tan(90^o - \gamma *)] \quad (44).$$

B) Shear in the forelimb

The angular parallel shear in the forelimb can be expressed as (Figure 22):

$$\psi = \tan^{-1}[\tan(90^o - x) + \frac{t' \tan(90^o - \gamma ** - \delta + x)}{t}] \quad (45).$$

In the forelimb, using trigonometric relationships, we know that (Figure 22):

$$\frac{t}{\sin x} = \frac{t'}{\sin(\gamma ** + \delta - x)} \quad (46).$$

Using the formulas of addition of two angles this equation can be expressed as:

$$t\sin(\gamma^{**}+\delta)\cos x - t\cos(\gamma^{**}+\delta)\sin x = t'\sin x \qquad (47),$$

and simplifying:

$$\cot x = \frac{t' + t\cos(\gamma^{**}+\delta)}{t\sin(\gamma^{**}+\delta)} \qquad (48).$$

Replacing γ** from equation (43) into equation (48) and rearranging we obtain the following equation:

$$x = \tan^{-1}[\frac{t\sin(\sin^{-1}[t'/t\sin\gamma^*]+\delta)}{t'+t\cos(\sin^{-1}[t'/t\sin\gamma^*]+\delta)}] \qquad (49).$$

Replacing equations (43) and (49) into general equation (45) the angular shear in the forelimb can be also expressed as a function of the thickness, δ and γ*:

$$\psi = \tan^{-1}[\tan(90^o - \tan^{-1}[\frac{t\sin(\sin^{-1}[t'/t\sin\gamma^*]+\delta)}{t'+t\cos(\sin^{-1}[t'/t\sin\gamma^*]+\delta)}]) + \frac{t'}{t}\tan(90^o - \sin^{-1}[t'/t\sin\gamma^*] - \delta + \tan^{-1}[\frac{t\sin(\sin^{-1}[t'/t\sin\gamma^*]+\delta)}{t'+t\cos(\sin^{-1}[t'/t\sin\gamma^*]+\delta}])] \qquad (50).$$

C) Shear in the backlimb

The angular shear in the backlimb is expressed by (Figure 22):

$$\psi = \tan^{-1}[2\tan(\delta/2 - \gamma^*/2)] \qquad (51).$$

By subtracting equation (40) to equation (45) plus equation (51), equation (17) is obtained.

To plot the total angular shear for a detachment fold without flat top and forelimb thinning or thickening (Figure 16) equation (17) has to be expressed as a function of the thickness, δ and γ*. This is obtained from equation (50) minus equation (44) plus equation (52):

$$\psi = \tan^{-1}[\tan(90^o - \tan^{-1}[\frac{t\sin(\sin^{-1}[t'/t\sin\gamma^*] + \delta)}{t' + t\cos(\sin^{-1}[t'/t\sin\gamma^*] + \delta)}]) +$$

$$\frac{t'}{t} \tan(90^o - \sin^{-1}[t'/t\sin\gamma^*] - \delta + \tan^{-1}[\frac{t\sin(\sin^{-1}[t'/t\sin\gamma^*] + \delta)}{t' + t\cos(\sin^{-1}[t'/t\sin\gamma^*] + \delta}])] -$$

$$\tan^{-1}[\frac{t'\tan(90^o - (\sin^{-1}[t'/t\sin\gamma^*]))}{t} + \tan(90^o - \gamma^*)] +$$

$$\tan^{-1}[2\tan(\delta/2 - \gamma^*/2)] \qquad (52).$$